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INSTITUTO DE INGENIERÍA ENERGÉTICA
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Research Publications

Semi-Explicit Method for

Wall Temperature Linked Equations (SEWTLE)

A General Finite Volume Technique

for the Calculation of Complex Heat Exchangers

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Abstract

An in-depth analysis has been carried out on the discretization of a Heat Exchanger, the discretization of the governing equations, and the solution strategy for the resulting system of equations. An iterative global method to perform the analysis of HE is proposed, called **SEWTLE.** This method provides flexibility to any flow arrangement or geometry, consideration of multiple streams, local evaluation of properties, friction factor and heat transfer coefficient and is characterised by good accuracy, high robustness and fast computation time. In addition, three different numerical schemes for the discretization of the fluid and wall conservation equations at cell level have been studied and their advantages and disadvantages fully discussed.

Nomenclature

a	thermal conductance (W/K)	NTU	Number of Heat Transfer Units (-)
A	cross section area (m^2)	P	perimeter (m)
c_p	specific heat (J/kg K)	q	heat flux (W/m^2)
C_R	capacity ratio (-)	Q	heat (W)
h	heat transfer coefficient (W/m^2 K)	Rw	wall thermal resistance (m^2K/W)
k	conductivity (W/m K)	t	wall thickness (m)
k_{ji}	NTU_{ji} per length unit (m^{-1})	T	temperature (K)
m	mass flow rate (kg/s)	Tw	wall temperature (K)

U	overall heat transfer coefficient (W/m^2 K)
x, y	spatial co-ordinates (m)

Greek symbols

χ_i	NTU$_i$ per length unit (m^{-1})
$\delta x, \delta y$	distance between grid points (m)
Δ	increment
η_T	surface total efficiency
θ	non dimensional temperature $\theta = \frac{T - T_{ref}}{T^{in} - T_{ref}}$
∇	operator

Subscripts

E,W,N,S	east, west, north, south
i	fluid cell index
i, in	inlet
it	iteration number it
j	wall cell index
k	neighbour cell index
n_i	number of wall cells surrounding fluid cell i
o, out	outlet
ref	reference
w	wall
$1,2$	top and bottom fluid cells

1 Governing Equations

There is much literature concerning calculation methods for Heat Exchangers (HE). Most belong to what we could call classical analysis and design global methods, i.e. the Logarithmic Mean Temperature Difference (LMTD) method, the Effectiveness-Number of Transfer Units (ε-NTU) method, and so on. They are very well established and cover all types of HE, including multipass HE. However, in general, these methods are based on some simplification hypothesis and conditions: basically, steady flow, single phase flows, constant properties and heat transfer throughout the HE, negligible longitudinal wall heat conduction, uniform distribution of the flows, and so on. These methods have traditionally been the basis for the rating and design of HE, and provide, in general, good results. However, for more complex cases, for instance HE with two-phase flow processes, air dehumidification or variable fluid properties, these methods are not accurate enough. Additionally, nowadays, the requirement for optimisation of the designs is a general demand, and this optimisation requires, on the one hand, a more accurate calculation, and on the other hand, more information about the evolution of the fluids inside the HE.

Of course, consideration of the above-mentioned phenomena in the HE calculation requires a division of the HE into cells, and the use of a numerical scheme to discretize the equations, and then, the solution of the resulting system of equations (linear or non linear). There is also extensive literature concerning the solution to the above-mentioned problems by discretization techniques. However, most of these techniques, although probably general in nature, have been targeted as the solution to the global analysis of HE including only a single particular effect, for instance, the consideration of the longitudinal heat conduction ([1], [2]) or the non-uniformity of the inlet fluid flow ([3], [4]), and only a few studies have been devoted to discussing and developing a general numerical formulation (for instance [5]).

When reviewing the local form of discretized equations, one concludes that most of the forms are similar. But upon a deeper analysis, it is shown that they are not the same, and that large differences could appear in their practical use. The two main streams found in the numerical solution of Engineering problems, i.e. Finite Volumes and Finite Elements can also be found in this field; see for instance [6] and [7].

The aim of this paper is to identify and discuss possible elementary discretizations leading to different numerical features, provided that they are as general as possible, in terms of multistream consideration, future extension to two phase flow, air dehumidification, unsteady flow, and so on. This study is limited to finite volume discretizations; a comparison with possible finite element formulations is left for a future work.

2 Governing Equations

As explained above, the present study is devoted to the calculation of HE of any flow arrangement and geometry, steady conditions, single-phase flows, and longitudinal conduction along the wall. Also, the possibility that a fluid stream can be surrounded by n separating walls has been considered in order to deal with HE with multiple streams (greater than 2). Figure 1 shows the described general situation. Given the assumptions above, the governing equations for both fluids and walls are

$$\dot{m}_i\, c_{pi}\, dT_i = \sum_{j=1}^{n_i} q_{ji} P_{ji}\, dx_i \tag{1}$$

$$P_{ji} = \frac{dA_{ji}}{dx_i} \tag{2}$$

$$q_{ji} = U_{ji}\left(Tw_j - T_i\right) \tag{3}$$

$$U_{ji} = \frac{1/A_{ji}}{Rw_{ji} + 1/A_{Tji}\eta_{Tji}h_{ji}}$$

$$Rw_{ji} = \frac{t_j/2}{A_{ji}k_j}$$

$$\nabla\left(k_j t_j \nabla Tw_j\right) + \sum_{i=1}^{2} q_{ji} = 0 \tag{4}$$

where any piece of wall *j* (wall cell *j*) separates two fluid cells *i=1,2*; and any piece of fluid (fluid cell) *i* is in contact with n_i wall cells $j=1,n_i$; q_{ji} is the heat flux transferred from the wall cell *j* to the fluid cell *i*; dx_i is the spatial differential in the forward direction of the *i* fluid in the *i* fluid cell; A_{ji} is the projected area (on the wall plane) of the wall cell *j* in contact with the *i* fluid cell; P_{ji} is the perimeter of the wall cell *j* wetted by fluid *i*, 'corresponding to the projected area', in the *i* direction; A_{Tji} and η_{Tji} are the total heat transfer area from the wall cell *j* to fluid *i* and its corresponding efficiency; t_j and k_j are the thickness of the *j* wall cell and its thermal conductivity (these variables could be defined with different values for both x and y directions, so that the actual thermal conductance of the wall ensemble (plate and fins) can be properly represented in both directions); and Rw_{ji} is the thermal resistance of half thickness of the wall for the heat transferred to fluid *i*. The other variables are defined in the section on Nomenclature.

In addition to (1) to (4), one could write the momentum conservation equation, which, together with the mass conservation equation, provides the system of equations to calculate the evolution of pressure and velocity. For steady and single-phase flows, mass flow rate is constant, and the energy equation is not coupled with the other equations and can be solved independently. First, the energy equation is solved and temperature is obtained. Then, the evolution of velocity and pressure is calculated. The present paper deals with the integration of the described system of equations for the temperature. However, the possibility of using the same discretization for more complex situations has always been considered. The extension of the scheme presented in this paper to evaporation, condensation, and air dehumidification can be found in [8] and [9].

3 Boundary Conditions

Boundary conditions (BC) for this problem are imposed at the inlet face of the streams, in which the fluid temperature and velocity distributions are known. Also, if the heat transferred to the surroundings from the wall edges can be considered negligible, one must impose the condition that the derivative of the wall temperature at the end sides of the HE, perpendicular to the edge, is null. Therefore, a Dirichlet type condition is imposed to the inlet fluid temperatures, and a Neumann type condition to the wall temperatures all around the exterior edge. Finally, the closing walls at the end sides of the heat exchanger are normally well-insulated so that heat transferred to the surroundings can be assumed negligible, and the wall assumed adiabatic.

4 Solution method

According to Smith [10], if a solution of the wall and fluid temperatures could be found for the flow inlet faces, the chances of finding a solution to the complete problem would be improved, and possibly an analytical solution could be obtained in some cases. He also suggests that a third order ordinary differential equation for each fluid could be obtained for each inlet face if the wall temperature at the origin were evaluated from knowledge of the local heat transfer coefficients, and if no longitudinal conduction were considered. However, the resulting differential equations turn out to be very ‘stiff’ and thus no effective solution is found.

Therefore, the solution must be found, in the general case, by a numerical procedure. Additionally, if one intends to take into account the local variation of the properties and the heat transfer coefficients, then the numerical way is definitively the only possibility to obtain a solution for the problem.

As commented in the introduction, the two main current tendencies can be applied to manipulate and discretize the equation: the Finite Element Method (FEM) and the Finite Volume Method (FVM). This paper deals with the application of the latter technique to the solution of the problem. A comparison with FEM is left for future work, although, a priori, it is the authors' opinion that the FVM is the best method for the problem studied and for more complex cases.

Concerning the type of mathematical problem which is to be solved, it consists of a combination of a system of elliptic Partial Differential Equations (PDE) (4) for the wall temperatures, with the particularity that they include a non linear source term (heat fluxes to the fluids), with Neumann type BC, together with a system of Ordinary Differential Equations (ODE)(1) for the fluid temperatures, with Dirichlet type BC. The resulting system of equations being coupled through the main variables (temperatures), and, perhaps, including strong non linearities specially introduced by the heat transfer coefficients (e.g. the sharp discontinuity of the heat transfer coefficient for an air flow depending on the existence or non existence of dehumidification, a fact which is dependent on the wall temperature for given air conditions).

Most of the solution methods employed are based on the elimination of the wall temperature from the stated system of equations when longitudinal conduction is negligible and then, equation (3) becomes a linear equation relating wall temperature with neighbour fluid temperatures. However, in the general case at hand, in which the properties and the heat transfer coefficients could be highly dependent on temperature and pressure, equation (3) is not linear at all, and the elimination of the wall temperature is not possible. Therefore, in the following, the decision is made to include the calculation of wall temperatures in the numerical procedure to be developed. This has the additional advantage that, in this case, fluid temperatures are only coupled with the global wall temperature field through equation (3), but not with the temperature of the other fluids, so that the problem becomes, in general, one of finding the solution to the wall temperature field. Then, the solution for the fluids can be found from the integration of equation (1). For this reason the authors propose naming this system of equations '**Wall Temperature Linked Equations (WTLE)'**.

5 Cell discretization

For the purpose of numerical calculation, the heat exchanger must be discretized in cells, which means that both fluid streams and separating walls must be discretized in a number of cells. The only condition is that all the fluid flows, and the wall, must be covered without any overlapping. In order to keep calculation time low, the assumption that fluid flow is one dimensional along a cell will be adopted. Therefore, the discretization of the fluid field must be done in such a way that real flow is approached as accurately as possible, as a series of one dimensional paths of 1D-cells. Fortunately, this is quite true for a large number of HE. Fluid coming out from different fluid cells can be adequately mixed at particular sections of the HE, if this is considered to happen in the real HE. This consideration does not affect the basic procedure to be explained, so that, in the following, when the fluid enters one of the 1D paths, it is considered to flow without any mixing with the other paths.

No restriction on the cell size is necessary, given that any separating wall cell between two fluid cells must be coincident with the shared side of the fluid cells.

Although the above stated conditions are not restrictive, in practice, the difficulties in arranging the flows normally lead to, at the cell level, either cross flow or parallel flow (co-current or countercurrent). This situation is represented in Figure 2. Parts a)[1] and b) show typical arrangements with plane walls. Internal walls can be always considered as fins attached to the separating walls. The possible situation depicted in Figure 2, parts c) and d) represent a parallel flow arrangement but, in these cases, thermal exchange among three different fluids is considered. Then, besides the main separating wall cells, additional wall cells must be considered if conduction among the separating wall cells is considered as not negligible; these wall cells are

[1] Notice that in Fig. 2, part a), $A_{j1} = A_{j2} = \Delta x_1 \, \Delta x_2$, $P_{j1} = \Delta x_2$ and $P_{j2} = \Delta x_1$.

square base prisms in c) and triangular base in d). The consideration of these prisms does not introduce any special difficulty to the numerical method employed below for the wall temperature calculation, so in the following, only situations a) and b) will be considered.

A first decision now must be made, i.e. how to define the control volumes for the fluids and for the wall. As a general rule, differential equations, such as (1), in which the main term contains a first order spatial derivative, require the use of the variable at the entrance and at the outlet of the control volume. In this way, the first derivative can be efficiently approximated by the corresponding first order finite difference expression linking the well-defined values of the variable at both ends. The integration then of the temperature difference appearing in (3) allows for speculation, as shown later.

On the other hand, Equation (4), governing the wall temperature field, includes second order partial spatial derivatives, so its integration normally requires a centred control volume with a value of temperature defined at its centre. In that way, the second order difference discretization can be efficiently defined.

Several different options can now be adopted to fit both discretizations for fluids and wall, for the involved elementary heat exchange cells. Figure 3 shows the two possible options for countercurrent parallel flow. In a) the wall cell is coincident with the piece of wall shared between two fluid cells, and therefore, wall temperature is defined at the centre of the wall cell and at the middle of both fluid paths. In b) the wall cell is staggered in regard to the fluid cells so that wall temperature is known at the same location as fluid temperatures.

On the other hand, Fig.4 shows several possible definitions for cross flow. Part a) shows again the case in which the wall cell is coincident with the piece of wall shared between two fluid cells, and therefore, wall temperature is defined at the centre of the wall cell and at the middle of both fluid paths. Part b) shows a definition of staggered mesh in such a way that wall temperature is known at the same location as fluid temperatures. However, then, wall cells present overlap, thus making the integration of (4) difficult. Furthermore, in the absence of longitudinal conduction, it becomes clear that an excessive de-coupling between directions x and y appears, which will probably lead to great numerical difficulties in finding the solution. Finally, part c) shows a staggered mesh, without overlapping.

It should be pointed out that this first step towards the discretization of the domain is essential to the success of the numerical technique to be developed. The staggered arrangements, shown in Fig. 3 part b) and Fig. 4 part b), present an important disadvantage in comparison with the centred option, i.e. they do not easily satisfy the Neumann boundary condition posed at the edge of the plates. In contrast, the centred option is perfectly suited for that task. Additionally, the advantage presented by the coincidence of the wall and fluid temperatures at the same location is lost in the case described in Fig. 4 part c) requiring interpolation between neighbouring nodes. On the other hand, integration of (4) over the wall cell must be based on the fluid temperatures known at the wall cell edges. But it is assumed that no mixing occurs inside the 1D flow cells. Therefore, a consistent definition of the fluid over and under a wall cell is not trivial at all.

The staggered arrangements depicted in Fig. 3 part b), and Fig. 4 part c) are normally used for FEM implementation, as for instance in [2] and [4], leading to good results. FEM can exploit the advantages of the discussed arrangement while overcoming some of the mentioned difficulties. However, the staggered arrangement clearly does not seem suitable for application of a FVM formulation.

Alternatively, with the centred option, the integration of equation (1) for the fluids would mainly involve the wall temperature at the centre of the wall cell. Some influence of the neighbour cells could be considered depending on the final selected scheme but the main influence will be always that of the central point. Moreover, the integration of (4) does not present any problem from the point of view of the fluid temperatures since the fluid paths are consistently defined over the wall cell. These features make the centred option much more adequate for the numerical solution of the problem. Therefore, the centred wall cell, as depicted in Fig.3 part a) and Fig. 4 part a) will be used in the following.

6 Numerical Scheme

Following the concept of Wall Temperature Linked Equations (WTLE) introduced earlier, the decision is made to de-couple the calculation of the wall temperature field from that of the temperature of the fluids.

The integration of equation (4) throughout the walls, by assuming that fluid temperatures are known at the fluid nodes, is considered as one step of the numerical procedure. The discretization of this equation does not offer any special difficulty, except for the estimation of the integral of the heat transferred to both fluids in contact with the considered piece of wall (equation (3)). This integration must be consistent with the integration of the coincident terms of the fluid energy equation (1). The discretization of the Laplacian operator in equation (4) can be made by a classical finite difference (finite volume) approach. For the case shown in Fig. 4 part a), the discretization of the first term of equation (4) would lead to:

$$a_j Tw_j - \sum_{k=W,E,N,S} a_j^k Tw_j^k = -\int_0^{\Delta x} \Delta y U_{j1}(Tw - T_1)dx - \int_0^{\Delta y} \Delta x U_{j2}(Tw - T_2)dy \quad [2] \qquad (5)$$

with: $a_j^W = \frac{k_w \Delta y}{\delta x_W} t_j, \quad a_j^E = \frac{k_E \Delta y}{\delta x_E} t_j, \quad a_j^N = \frac{k_N \Delta x}{\delta y_N} t_j, \quad a_j^S = \frac{k_S \Delta x}{\delta y_S} t_j, \quad a_j = \sum_k a_j^k$

Then, if the discretization of the integrals of the heat transferred to both fluids in contact with the considered piece of wall (right-hand side of equation (5)) is based on a linear function relating the values of the fluid and wall temperatures at the neighbouring points, equation (5) leads to a linear equation for the wall temperature at every cell node, involving, the temperatures of the fluids in contact with both cell surfaces.

On the other hand, equation (1) is a first order differential equation to be integrated along the direction of the flow path. Consequently, it can be discretized in a number of different ways depending on the approach adopted for the wall temperature distribution and the fluid temperature

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[2] Notice that for this case, $A_{ji} = \Delta x \Delta y, \; P_{j1} = \Delta y, \; P_{j2} = \Delta x, \; Rw_{ji} = Rw_{ij} = \frac{t/2}{\Delta x \Delta y \, k_j}$

evolution, provided always that the evaluation of the integrals appearing at the right-hand side of equation (1) are consistent with the coincident terms in (5).

A first classification could be based upon how many wall temperatures are taken into consideration in the discretized equation, i.e. 1 wall temperature (value at the cell centre), or several: central point and neighbours (3 points in parallel flow and 5 points in cross flow).

A similar comment can be made about the number of fluid temperatures involved in the integration of equation (1). Equation (1), integrated along a fluid path, can be interpreted as an initial value problem for a given inlet temperature and a given wall temperature distribution. Hence, only temperatures upstream should be considered in the numerical scheme. Two main choices are then possible: 1) the use of the temperature at the cell inlet as the inlet boundary, or, 2) the extrapolation of upstream information, i.e. multipoint methods.

In the following, only the first choice will be analysed, given that the consideration of multiple points would not influence the main conclusions drawn from the present study.

Three different schemes are to be presented and discussed. They are schematically described in Figure 5.

- **Constant Wall Temperature** (**CWT**) will assume that along a fluid cell the wall temperature can be considered as uniform and equal to the value at the central point of the wall cell. This approach should be equivalent to supposing a stepwise profile for the temperature at the heat exchanger walls.
- **Linear Fluid Temperature Variation** (**LFTV**) is based on temperature average and the assumption that the fluid temperatures along the fluid paths have a piecewise distribution (see Fig. 5 part b).
- **High Order Differential Scheme** (**HOD**) is based on the Runge Kutta method for integration of ODE. This scheme has been included in the study to assess what could be expected from a higher order spatial discretization and higher accuracy.

In the following sections, the development of the commented schemes is presented for the general case of a fluid cell surrounded by n_i separating walls. We include the expressions of the outgoing temperature of the fluid (fluid cell) i in contact with n_i wall cells $j=1,2,...,n_i$ coming from the integration of equations (1) to (3), the heat transferred from the wall cell *j* to the fluid cell *i*, equation (3) and right-hand side of equation (5). Finally, the wall temperature at the centre of the wall cell *j*, which comes from the discretization of equation (5) is also included.

6.1 Constant Wall Temperature (CWT)

We will analytically carry out the integration of equations (1) and (3), which can be combined in the following way,

$$\frac{dT_i}{dx_i} = \sum_{j=1}^{n_i} k_{ji}(Tw_j - T_i) \tag{6}$$

with: $k_{ji} = \frac{P_{ji}U_{ji}}{\dot{m}_i c_{pi}}$

In this case (CWT), we assume that, along a fluid cell, the wall temperature is uniform and equal to the value at the central point of the wall cell. Thus, we can integrate the previous equation along a fluid cell in the following way:

$$\int_{T_i^{in}}^{T_i^{out}} \frac{dT_i}{\sum_{j=1}^{n_i} k_{ji}\,(Tw_j - T_i)} = \int_0^{\Delta x_i} dx_i$$

and we obtain the outgoing temperature of the fluid (at fluid cell *i*):

$$T_i^{out} = \frac{\sum_{j=1}^{n_i} k_{ji}\, Tw_j}{\chi_i}\left(1 - \exp\left(-\chi_i \Delta x_i\right)\right) + T_i^{in} \exp\left(-\chi_i \Delta x_i\right) \tag{7}$$

where the value of χ_i is given by[3]: $\chi_i = \sum_{j=1}^{n_i} k_{ji}$

We also know the distribution of the fluid temperature field along the fluid cell, so we can evaluate the expression of the heat transferred from the wall cell *j* to the fluid cell *i.*

$$Q_{ji} = \int_0^{\Delta x_i} U_{ji} P_{ji} (Tw_j - T_i) dx_i \tag{8}$$

where the dependence of T_i on the coordinate x_i is given by equation (7). Then, replacing Δx_i by the value of x_i, and through the integration of (8), we finally obtain

$$Q_{ji} = \frac{-U_{ji} P_{ji} \Delta x_i}{1 - \exp(-\chi_i \Delta x_i)} \left\{ T_i^{out} \left(1 - \frac{1 - \exp(-\chi_i \Delta x_i)}{\chi_i \Delta x_i} \right) + T_i^{in} \left(\frac{1 - \exp(-2\chi_i \Delta x_i)}{\chi_i \Delta x_i} - \exp(-\chi_i \Delta x_i) \right) - Tw_j \right\} \tag{9}$$

The last step is to derive the wall temperature at the centre of the wall cell j, from equation (5). Using the expression (9) for the integrals in equation (5), we obtain the value of Tw_j:

$$Tw_j = \frac{1}{a_j + \Delta x_1 \Delta x_2 \sum_{s=1}^{2} \frac{U_{js}}{1 - \exp(-\chi_s \Delta x_s)}} \cdot$$

$$\left\{ \sum_{k=W,E,N,S} a_j^k Tw_j^k + \Delta x_1 \Delta x_2 \sum_{s=1}^{2} \frac{U_{js}}{1 - \exp(-\chi_s \Delta x_s)} \left[T_s^{out} \left(1 - \frac{1 - \exp(-\chi_s \Delta x_s)}{\chi_s \Delta x_s} \right) + T_s^{in} \left(\frac{1 - \exp(-2\chi_s \Delta x_s)}{\chi_s \Delta x_s} - \exp(-\chi_s \Delta x_s) \right) \right] \right\}$$

(10)

6.2 Linear Fluid Temperature Variation (LFTV)

In this case we consider a linear fluid temperature variation. The first step involves the evaluation of the outgoing temperature of the fluid (fluid cell) i via the integration of equation (6).

$$\int_{T_i^{in}}^{T_i^{out}} dT_i = \int_0^{\Delta x_i} \sum_{j=1}^{n_i} k_{ji} (Tw_j - T_i) dx_i = \sum_{j=1}^{n_i} k_{ji} \overline{(Tw_j - T_i)} \Delta x_i \tag{11}$$

where $\overline{\alpha}$ stands for an averaged value of the quantity α.

In our approach for LFTV, we can write

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[3] Notice that $\chi_i \Delta x_i$ is the number of transfer units NTU_i of the fluid cell i with the surrounding wall cells n_i .

$$\overline{(Tw_j - T_i)} = \overline{Tw_j} - \overline{T_i} \approx Tw_j - \frac{T_i^{in} + T_i^{out}}{2}$$

Then, the outgoing temperature of the fluid becomes

$$T_i^{out} = \frac{(1 - 0.5\Delta x_i \chi_i) T_i^{in} + \Delta x_i \sum_{j=1}^{n_i} k_{ji}\, Tw_j}{(1 + 0.5\Delta x_i \chi_i)} \tag{12}$$

The second step involves the evaluation of the heat transferred from the wall cell *j* to the fluid cell *i,* following equation (8). Using the same approximation as in equation (11) we obtain

$$Q_{ji} = U_{ji} P_{ji} \left(Tw_j - \frac{T_i^{in} + T_i^{out}}{2} \right) \Delta x_i \tag{13}$$

Finally, using expression (13) for the integrals in equation (5), the wall temperature at the centre of the wall cell *j* becomes

$$T_{wj} = \frac{\sum_{k=W,E,N,S} a_j^k Tw_j^k + \sum_{s=1}^{2} U_{js} P_{js} \left(\frac{T_s^{in} + T_s^{out}}{2} \right) \Delta x_s}{a_j + \sum_{s=1}^{2} U_{js} P_{js} \Delta x_s} \tag{14}$$

6.3 Runge Kutta based High Order Differential Scheme (HOD)

In this section we integrate equation (6) using a 4-step Runge-Kutta method. If we use the following notation in eq. (6):

$$\frac{dT_i}{dx_i} = \sum_{j=1}^{n_i} k_{ji} (Tw_j - T_i) = f(T_i, Tw_1, Tw_2, \ldots, Tw_{ni})$$

We can evaluate T_i^{out} solving the first order ODE through the following steps:

$$T_i'^{(1)} = f(T_i^{in}, Tw_1^{in}, Tw_2^{in}, \ldots, Tw_{ni}^{in})$$

$$T_i^{(2)} = T_i^{in} + ½\, \Delta x_i\, T_i'^{(1)} \qquad T_i'^{(2)} = f(T_i^{(2)}, Tw_1, Tw_2, \ldots, Tw_{ni})$$

$T_i^{(3)} = T_i^{in} + \frac{1}{2}\,\Delta x_i\; T_i'^{(2)}$ $\qquad$ $T_i'^{(3)} = f(T_i^{(3)}, Tw_1, Tw_2, \ldots, Tw_{ni})$

$T_i^{(4)} = T_i^{in} + \frac{1}{2}\,\Delta x_i\; T_i'^{(3)}$ $\qquad$ $T_i'^{(4)} = f(T_i^{(4)}, Tw_1^{out}, Tw_2^{out}, \ldots, Tw_{ni}^{out})$

$T_i^{out} = T_i^{in} + \Delta x_i\; (1/6\; T_i'^{(1)} + 1/3\; T_i'^{(2)} + 1/3\; T_i'^{(3)} + 1/6\; T_i'^{(4)})$

Following this method, and after a long algebra, the following expression for T_i^{out} can be obtained:

$$T_i^{out} = A_i T_i^{in} + B_i \Delta x_i \sum_{j=1}^{n_i} k_{ji} Tw_j^{in,i} + C_i \Delta x_i \sum_{j=1}^{n_i} k_{ji} Tw_j^{out,i} + D_i \Delta x_i \sum_{j=1}^{n_i} k_{ji} Tw_j \qquad (15)$$

where

$$A_i = 1 - NTU_i + \frac{1}{2} NTU_i^2 - \frac{1}{6} NTU_i^3 + \frac{1}{24} NTU_i^4$$

$$B_i = \frac{1}{6} - \frac{1}{6} NTU_i + \frac{1}{12} NTU_i^2 - \frac{1}{24} NTU_i^3$$

$$C_i = \frac{1}{6}$$

$$D_i = \frac{2}{3} - \frac{1}{3} NTU_i + \frac{1}{12} NTU_i^2$$

and

$$NTU_i = \Delta x_i \sum_{j=1}^{n_i} k_{ji} = \sum_{j=1}^{n_i} \frac{P_{ji} U_{ji} \Delta x_i}{\dot{m}_i c_{pi}}$$

in which, NTU_i is the number of transfer units of fluid cell *i*, and $Tw_j^{in,i}$, and $Tw_j^{out,i}$ represent the temperature of the wall cell *j* at the inlet and outlet sections of fluid *i*. For example, in Fig. 4 part a; for fluid flow 1: $Tw_j^{in,1} = (Tw_j + Tw_j^W)/2$, and, $Tw_j^{out,1} = (Tw_j + Tw_j^E)/2$

The second step involves the evaluation of the heat transferred from the wall cell *j* to the fluid cell *i*. Using the total heat transferred to the fluid cell *i*, which is given by:

$$Q_i = \dot{m}_i\, c_{pi} \left(T_i^{out} - T_i^{in}\right)$$

and, since $A_i + NTU_i(B_i + C_i + D_i) = 1$, we obtain

$$Q_i = \dot{m}_i c_{pi}\left(T_i^{out} - T_i^{in}\right) = \dot{m}_i c_{pi}\Delta x_i \sum_{j=1}^{n_i} k_{ji}\left(B_i\left(Tw_j^{in,i} - T_i^{in}\right) + C_i\left(Tw_j^{out,i} - T_i^{in}\right) + D_i\left(Tw_j - T_i^{in}\right)\right)$$

Thus, the contribution of wall *j* to the heat transferred to fluid cell *i* is[4]:

$$Q_{ji} = \dot{m}_i c_{pi}\Delta x_i k_{ji}\left(B_i\left(Tw_j^{in,i} - T_i^{in}\right) + C_i\left(Tw_j^{out,i} - T_i^{in}\right) + D_i\left(Tw_j - T_i^{in}\right)\right) \qquad (16)$$

Finally, from equation (5) we obtain:

$$Tw_j = \frac{1}{a_j + \dot{m}_i c_{pi}\sum_{s=1}^{2} k_{js} D_s \Delta x_s} \cdot$$

$$\left\{\sum_{k=W,E,N,S} a_j^k Tw_j^k - \dot{m}_i c_{pi}\sum_{s=1}^{2} k_{js}\Delta x_s\left(B_s\left(Tw_j^{in,s} - T_s^{in}\right) + C_s\left(Tw_j^{out,s} - T_s^{in}\right) - D_s T_s^{in}\right)\right\} \qquad (17)$$

6.4 Basic differences among the three developed schemes

Before continuing, it is interesting to formally compare the three developed schemes in order to assess the main differences among them. Let us compare the expressions obtained for the calculation of the outgoing fluid temperature for the simplest case in which the fluid cell only exchanges heat with one wall cell. In order to facilitate the comparison, let us also arrange them into non-dimensional form by defining the non dimensional temperature θ as:

$$\theta = \frac{T - T_{ref}}{T^{in} - T_{ref}}$$

where T_{ref} is a reference temperature. Then, equations (7), (12) and (15) can be respectively transformed into the following expressions:

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[4] Notice that in the CWT and LFTV schemes, Q_{ji} can also be expressed exclusively in terms of Tw_j and T_i, as we have obtained in equation (16).

CWT $$\frac{\theta^{out}}{\theta^{in}} = \left(1-\exp\left(-NTU\right)\right)\frac{\theta_w}{\theta^{in}} + \exp\left(-NTU\right)$$

LFTV $$\frac{\theta^{out}}{\theta^{in}} = \frac{NTU}{1+NTU/2}\,\frac{\theta_w}{\theta^{in}} + \frac{1-NTU/2}{1+NTU/2}$$

HOD $$\frac{\theta^{out}}{\theta^{in}} = NTU\left(B\frac{\theta_w^{in}}{\theta^{in}} + D\frac{\theta_w}{\theta^{in}} + C\frac{\theta_w^{out}}{\theta^{in}}\right) + A$$

providing the value of the relative fluid temperature variation θ^{out}/θ^{in} as a function of the relative wall temperature θ_w/θ^{in} and the NTU size of the cell.

The obtained formulas clearly indicate that the outlet temperature is a linear average between the fluid temperature at the inlet section θ^{in} and an averaged value of the wall temperature. This averaged value is the value of the temperature at the cell centre for both CWT and LFTV schemes θ_w, and a more complex numerical average for the HOD scheme, which weighs the influence of the wall temperature variation on the heat transferred to the fluid.

For the case in which the wall temperature can be assumed constant along the cell, the non-dimensional expression of the HOD scheme can be further condensed, finally leading to:

'HOD for the constant wall temperature case' $$\frac{\theta^{out}}{\theta^{in}} = \left(1-A\right)\frac{\theta_w}{\theta^{in}} + A$$

Since $A_i = 1 - NTU_i + \frac{1}{2}NTU_i^2 - \frac{1}{6}NTU_i^3 + \frac{1}{24}NTU_i^4$, the HOD average consists of the first terms of the Taylor expansion series over *NTU*=0 of the exponential term $\exp\left(-NTU\right)$ corresponding to the result of the CWT scheme, which for this particular situation (constant wall temperature) provides the exact solution. Additionally, the average coefficients of the CWT scheme are always positive, meaning that the solution will always be bounded, whereas the coefficient A in the HOD scheme becomes negative at *NTU*=2.785, and the coefficient $\left(1-NTU/2\right)/\left(1+NTU/2\right)$ in the LFTV scheme becomes negative at *NTU*=2.

Two obvious possible improvements of the schemes arise from the above presented analysis. The LFTV scheme turns out to be not bounded for high NTU. Therefore, a hybrid scheme could be defined so that the coefficient of θ^{in} never becomes negative. On the other hand, the exponential function of the CWT scheme could be substituted by a power-law approximation, in such a way that computation time of that scheme does not suffer the penalty of the exponential function calculation.

7 Solution of the system of equations

The global system of ODE and PDE posed by equations (1) to (4) has been transformed into a system of pseudo-linear equations: one equation for every wall cell, in which the temperature of the cell and those of the neighbouring points appear beside the fluid temperatures of the flows in contact with the wall, and one equation for every fluid cell, in which the outlet temperature is related to the inlet temperature and the temperatures of the wall cells surrounding it (e.g., equations (10) and (7) for the CWT scheme).

Considering the boundary conditions for both walls and fluids, the global system becomes closed, and the solution to the problem involves the solution of the posed system of linear equations. The solution could be found by any standard direct or iterative procedure. However, before deciding on the most adequate solution strategy, one must bear in mind, first, that the system of equations is really non-linear, since some of the coefficients are dependent on the solution, i.e. temperature field, pressure, quality. Secondly, one must take into account that the actual values of the coefficients are, for the same reason, unknown. Therefore, it does not seem advantageous to find the exact solution to the described system of equations, then recalculate the coefficients, and start again until convergence has been reached. The correct strategy seems to combine the iterative calculation of the solution to the system of equations with the continuous evaluation of the coefficients, in such a way, that both calculations progress together towards the solution to the non-linear problem.

The envisaged solution commented above consists of three steps.

1. One must make an initial estimate of the temperature of the walls. Of course, the closer the estimate is to the solution the faster the convergence to the solution is. However, this aspect is not critical, and for instance, an arithmetic average of the inlet fluid temperatures can be employed with good results.
2. One must make the calculation of the fluid temperature evolution along the flow paths, respectively, from equations (7), (12) or (15). This is an explicit procedure so that the calculation proceeds quite fast. The pressure evolution can also be calculated through the integration of the momentum equation in the same manner. In more complex cases, like condensation or evaporation, similar explicit equations can be developed for the calculation of the main flow variables (see [8] for details). The fluid properties, the heat transfer coefficient and the friction factor at every cell are calculated for averaged values of the fluid and flow parameters. For the first iteration, these calculations are performed at the cell inlet conditions, which are always known from the calculation of the previous cell. For subsequent iterations, the properties and coefficients are always estimated at the new calculated inlet variables plus the value of the correction obtained in the previous iteration for the variation of the variable, for instance, for the temperature

$$\overline{T}(it) = T^{in}(it) + \frac{T^{out}(it-1) - T^{in}(it-1)}{2}$$

   i.e. the variation of the significant variables is calculated at every cell and stored to be used in the next iteration to evaluate the new averaged value at the cell. This numerical strategy has proved to be very adequate, providing good accuracy and fast computation.
3. Once all the temperatures of the fluids are known at every point, one must solve the system of equations (5) to obtain the wall temperatures (equations (10), (14) and (17) for the analysed schemes). In the case that the longitudinal conduction is significant, the equations involve 5 temperatures and any standard solver can be used. Again, there is no advantage in obtaining the final solution of the system, but it is preferable to performing only a single iteration of the employed method to obtain a recalculated value of the wall temperature at every point. Notice,

that equation (5) contains the usual discretization term coming from the Laplacian operator, which involves the conduction along the wall. Yet this term is not the leading one in the equation, but rather a complementary small correction, in fact, negligible in most of the cases. The leading term in equation (5) is the average of the temperatures of both surrounding fluids (right-hand side of eq. (5)), coming from the balance of the heat exchanged between the fluids. In fact, this is the part of the equation, which will always be active. In general, it provides the condition to evaluate the cell wall temperature in the way shown. In the absence of longitudinal conduction, this equation becomes explicit. In the presence of longitudinal conduction, this factor highly strengths the diagonal dominant character of the system, thus making the iterative solution easier. Any standard method could be applied to the general case; however, given the iterative character of the global solution procedure, the simple Gauss – Seidel procedure leads to excellent results. The only exception, from the authors' point of view, is that it is better to start the global iterative procedure without including the longitudinal conduction term in equations (10), (14) and (17), thus supposing its effect negligible. Then, after some iterations (typically 10), to switch on the calculation of the longitudinal conduction term effect. The global convergence is then reached in a few further iterations. This strategy allows for a smooth and correct evaluation of the longitudinal conduction effect.

- Steps 2 to 3 are repeated until convergence is reached.

Notice that the described solution method only requires the sequential evaluation of, first the temperature of the fluids and second, of the temperature of the walls, but the numerical schemes employed for both calculation steps are explicit. Thus, computing time per iteration is short. This explicitness and the inspiration from Prof. Patankar's book [11] has led to naming the proposed method as **Semi-Explicit method for Wall Temperature Linked Equations**, **SEWTLE**.

Fluid properties, and factors and coefficients, can be easily calculated every iteration as described in step 2, or updated only when a significant change of the variables occurs at the cell, leading to a faster computation.

8 Comparison of results and discussion

The described schemes where programmed and debugged through an extensive comparison of results for the basic different flow arrangements and a number of capacity ratios for which the exact analytical solution is known. Also a comparison of results on the influence of the longitudinal conduction when it is not negligible was performed for the three basic flow arrangements with the results published in [2]. The obtained results were coincident in all the analysed cases, with the logic dependence on the employed number of cells. In general, the error in the estimation of the efficiency of the heat exchanger is very low. A computational grid with only 5 cells (or 5x5 in cross flow) provides an error lower than 0.5%. This error is easily reduced when the number of cells is increased.

A systematic study was then carried out in order to assess the performance of the developed schemes. The diagrams comparing the computing time and the calculation error (referred to the exact analytical solution) for the three methods were investigated for the three basic flow arrangements, for two levels of HE NTU (0.5 and 1.5) and two different capacity ratios $C_R = C_{min}/C_{max}$ (0.25 and 1). The entire study was performed for two different numbers of mesh cells, 5 and 20.

Figure 6 shows a sample of the results obtained with each of the methods studied, for a crossflow HE with NTU=1.5 and C_R= 0.25, calculated with 20X20 cells. The left part shows the evolution of the error in comparison with the number of performed iterations, whereas the right part illustrates the variation of the error in regard to calculation time.

Figures 7, 8 and 9 show, for all the studied cases, the final error of each method (after the convergence was reached) and the approximate time that each method would require to reach a 0.5 % error[5]. The results of the calculation with 5 cells are shown in the left column of the figure. The column at the right corresponds to the results obtained with 20 cells.

[5] In a few cases, some of the schemes were unable to reach an error lower than 0.5 %. In those cases, in order to complete the shown graphs, the plotted time has been defined higher than that corresponding to the other methods.

The interpretation of the obtained results requires a previous comment regarding the capabilities of each scheme to reach the exact solution of the analysed cases. Notice that for the co-current case with balanced flows, the solution of the wall temperature is the averaged value between the inlet fluid temperatures, and constant all along the HE. This makes the solution by the CWT scheme quite favourable. On the other hand, in the countercurrent case the solution of the fluid temperatures and of the wall temperature, for balanced flows, becomes linear, rendering the LFTV scheme very favourable in this case. In general, the solution would not belong to either of the theoretical cases so that both methods will become approximate. The comparison of results for the cross flow situation (Figure 9) is overall more meaningful.

In general, from the analysis of the results, it can be concluded that the HOD method consistently leads to the smallest error. However, the comparison between the error obtained with 5 and 20 cells for all the schemes allows for a rough estimation of the order of accuracy, which turns out to be approximately 2 for all of them. Surprisingly, the HOD scheme does not lead to an order of accuracy higher than the other two schemes, but only to a smaller absolute error. This is due to the fact that the main advantage of the HOD scheme, as produced, is a lower discretization error in the wall temperature distribution, thanks to the use of linear interpolation of the wall temperature between neighbouring points for the integration of the heat transferred from a wall cell to a fluid cell. In contrast, it is important to point out that the error attained by the other two schemes is higher, but small enough to qualify them as efficient for both analysis and design.

From the computing time point of view, the first comment that should be argued is that, from the observation of equations (7) to (10), (12) to (14) and (15) to (17), it becomes evident that the fastest scheme is the LFTV one. It involves the lowest number of operations and they are simple. The CWT also requires a reduced number of operations, but it requires the evaluation of the exponential function which is comparatively much more time consuming. The number of operations required for the HOD per iteration is not much higher. However, if the evaluation of properties and the heat transfer coefficient is required at every iteration, then one must take into account that the evaluation of A, B, C and D coefficients requires a considerable computing time. That evaluation was only performed once for the study presented in Figures 6 to 9.

On the other hand, when the schemes are applied to the solution of a given HE problem, the calculation time of interest will be the time they require to provide a reasonably accurate solution, i.e. with error lower than a certain bound limit (for instance 0.5 %). That time will not only depend, of course, on the number of operations of the scheme per iteration, but also on the number of iterations required to approach the solution. This will depend on, first, how close the theoretical approach of the scheme is to the solution, and second, on the discretization order of the scheme.

It is very difficult to comparatively assess the calculation time performance of the three schemes for the solution of real HE problems. However, it is most important to assess if the higher accuracy of the HOD allows for a decrease in the number of iterations required to obtain a good accuracy (for example, 0.5 % error) which could compensate its higher computing cost. From the inspection of Figures 7 to 9, it can be observed that only in a few cases the time required by the HOD becomes the shortest and that, in those cases, the difference is not very large. The situation depicted in Figure 6 becomes quite representative of the situation. As it can be observed in that figure, the number of iterations required to reach the solution is very similar for all three schemes. The final error obtained is smaller with the HOD scheme, but it is also reasonably low with the other two methods. And, concerning the calculation time, it becomes apparent in Figure 6 (right) that the fastest scheme is the LFTV.

Throughout the entire study, the LFTV scheme offers, overall, the fastest calculation. Taking into account, additionally, that the performed comparison was favourable to the HOD scheme since its coefficients were not updated, and that the maximum error of the LFTV scheme is small, the conclusion is drawn that it is probably the most efficient scheme from the computational point of view. Only in those cases in which a constant wall temperature distribution is expected to occur, in which case the CWT scheme would provide better results, the LFTV seems to be the most adequate scheme for the analysis and design of HE.

Furthermore, it should be said that the authors have extended the SEWTLE method presented in this paper, with the LFTV or CWT schemes, to more complex heat exchangers ([8] and [9]), and that their performance has always been excellent, only requiring between 10 to 15 iterations to reach the solution.

9 Conclusions

- An in-depth analysis has been carried out on the discretization of a Heat Exchanger, the discretization of the governing equations, and the solution strategy for the resulting system of equations.
- Three different schemes have been presented and fully compared.
- The **LFTV** Scheme (Linear Fluid Temperature Variation) is proposed as the most adequate for HE analysis and design.
- Finally, an iterative global method to perform the analysis of HE is proposed, called **SEWTLE** (Semi-Explicit Method for Wall Temperature Linked Equations). This method provides flexibility to any flow arrangement or geometry, consideration of multiple streams, local evaluation of properties, friction factor and heat transfer coefficient, but it also is easily extendable to more complex heat transfer processes, and is characterised by good accuracy, high robustness and fast computation time.

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FIGURE CAPTIONS:

Figure 1. Fluid path and surrounding wall cells

Figure 2. Sample of different fluid and wall situations: 1) Two fluids, a) Crossflow, b) Parallel flow (co-current or countercurrent); 2) 3 different fluids A, B and C, geometrical cases c) and d).

Figure 3. Wall discretization for parallel countercurrent flow: a) Centred, b) Staggered.

Figure 4. Wall discretization for crossflow: a) Centred, b) Staggered with overlapping, c) Staggered without overlapping.

Figure 5. Temperature distribution for the different analysed schemes a) CWT, b) LFTV, c) HOD for the countercurrent flow case.

Figure 6. Error evolution for a cross flow heat exchanger with C_R = 0.25 and NTU = 1.5. Results obtained with 20 x 20 mesh cells.

Figure 7. Final error and Time required to reach an error lower than 0.5 %, for a co-current heat exchanger with: C_R = 0.25 and 1; NTU = 0.5 and 1.5; and for 5 x 1 (left) and 20 x 1 (right) calculation meshes.

Figure 8. Final error and Time required to reach an error lower than 0.5 %, for a counter-current heat exchanger with: C_R = 0.25 and 1; NTU = 0.5 and 1.5; and for 5 x 1 (left) and 20 x 1 (right) calculation meshes.

Figure 9. Final error and Time required to reach an error lower than 0.5 %, for a cross-flow heat exchanger with: C_R = 0.25 and 1; NTU = 0.5 and 1.5; and for 5 x 5 (left) and 20 x 20 (right) calculation meshes.

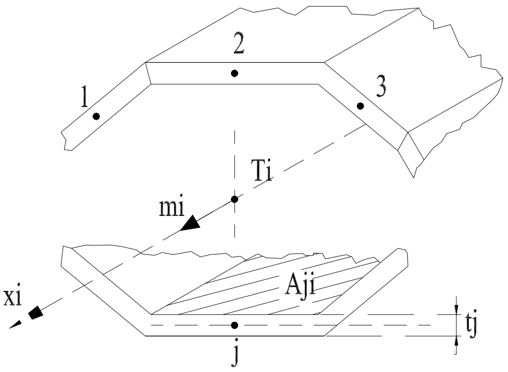
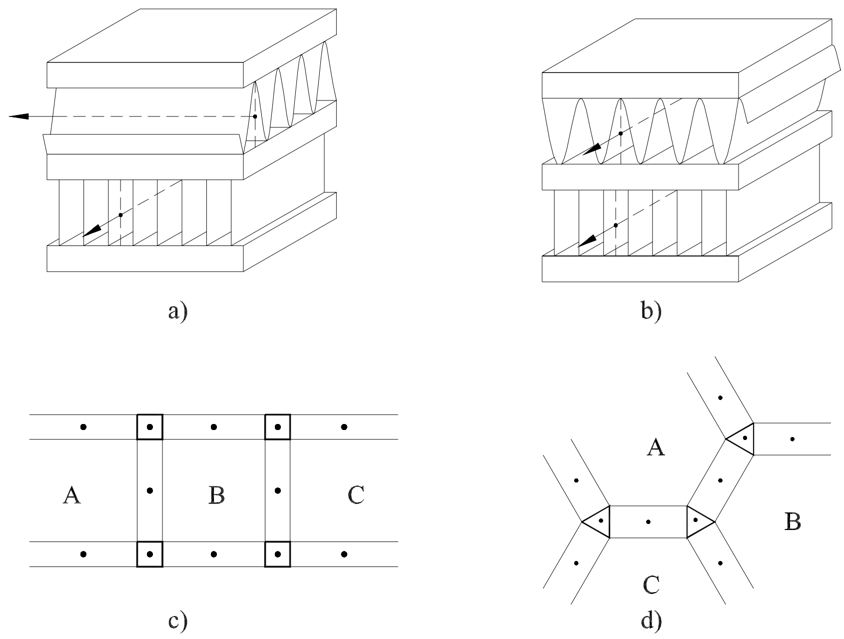


Figure 1. Fluid path and surrounding wall cells



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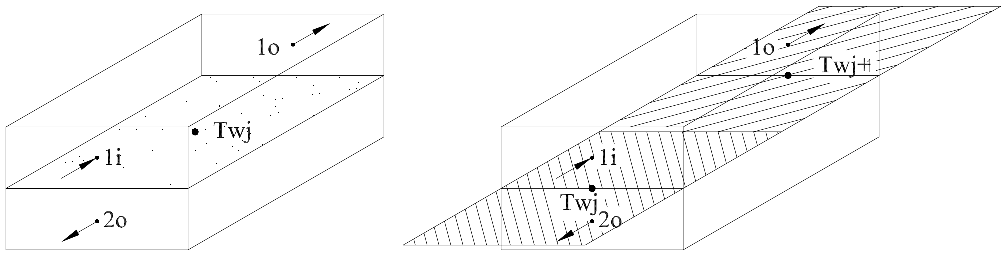

a) b)

Figure 3. Wall discretization for parallel countercurrent flow: a) Centred, b) Staggered.

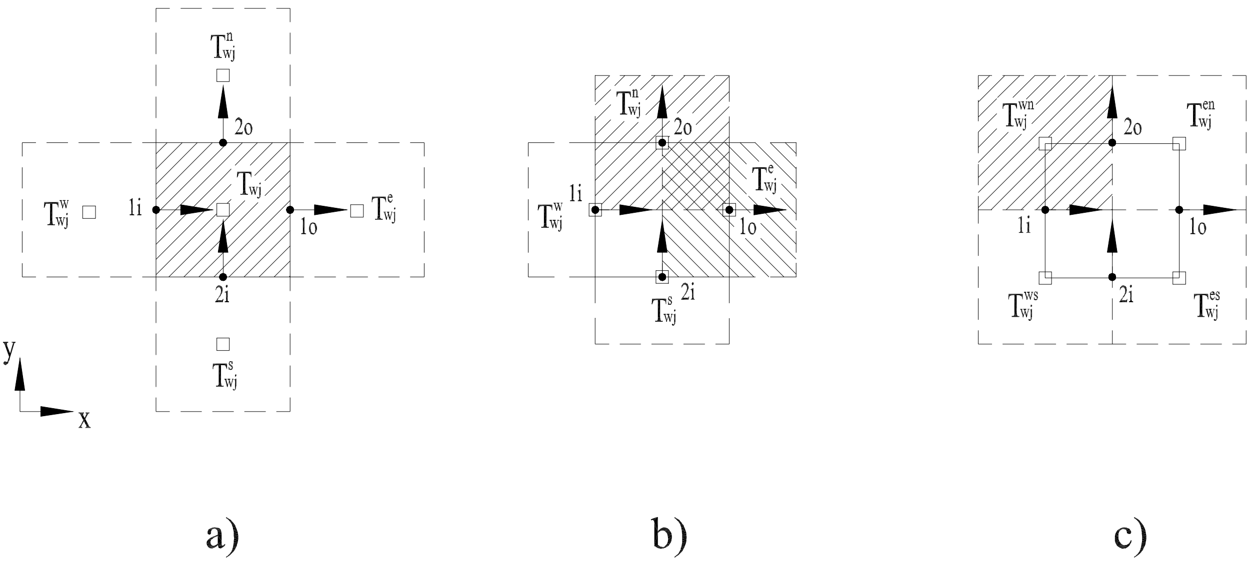

Figure 4. Wall discretization for crossflow: a) Centred, b) Staggered with overlapping, c) Staggered without overlapping.

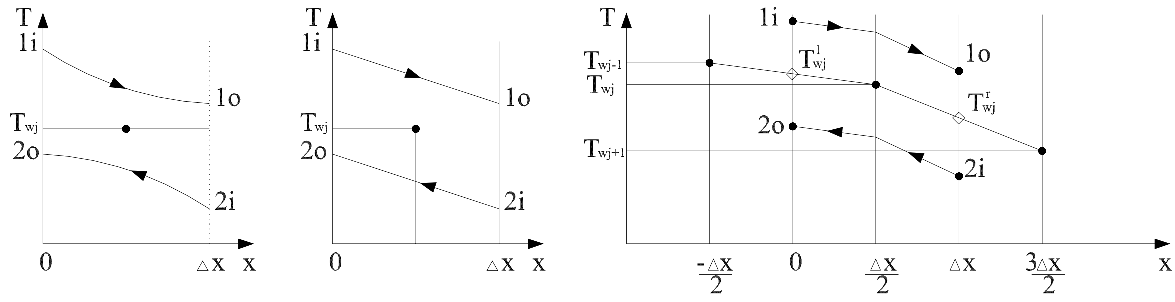

Figure 5. Temperature distribution for the different analysed schemes a) CWT, b) LFTV, c) HOD for the countercurrent flow case.

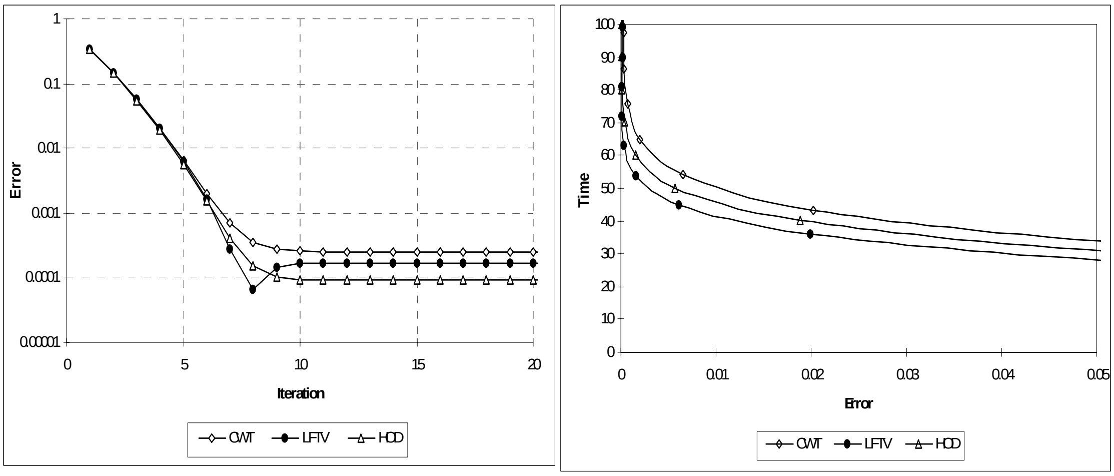

Figure 6. Error evolution for a cross flow heat exchanger with C_R = 0.25 and NTU = 1.5. Results obtained with 20 x 20 mesh cells.

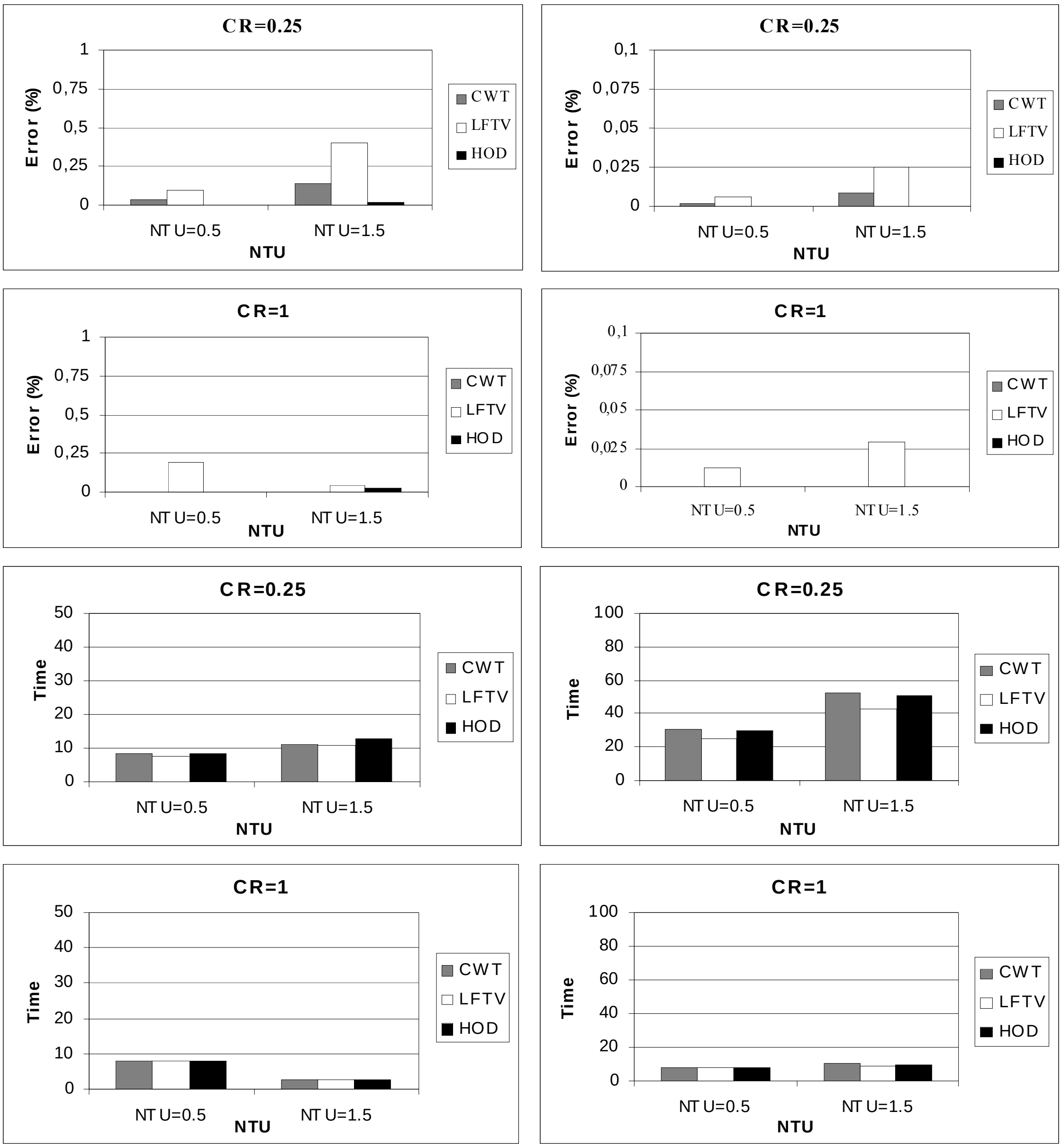

Figure 7. Final error and Time required to reach an error lower than 0.5 %, for a co-current heat exchanger with: C_R = 0.25 and 1; NTU = 0.5 and 1.5; and for 5 x 1 (left) and 20 x 1 (right) calculation meshes.

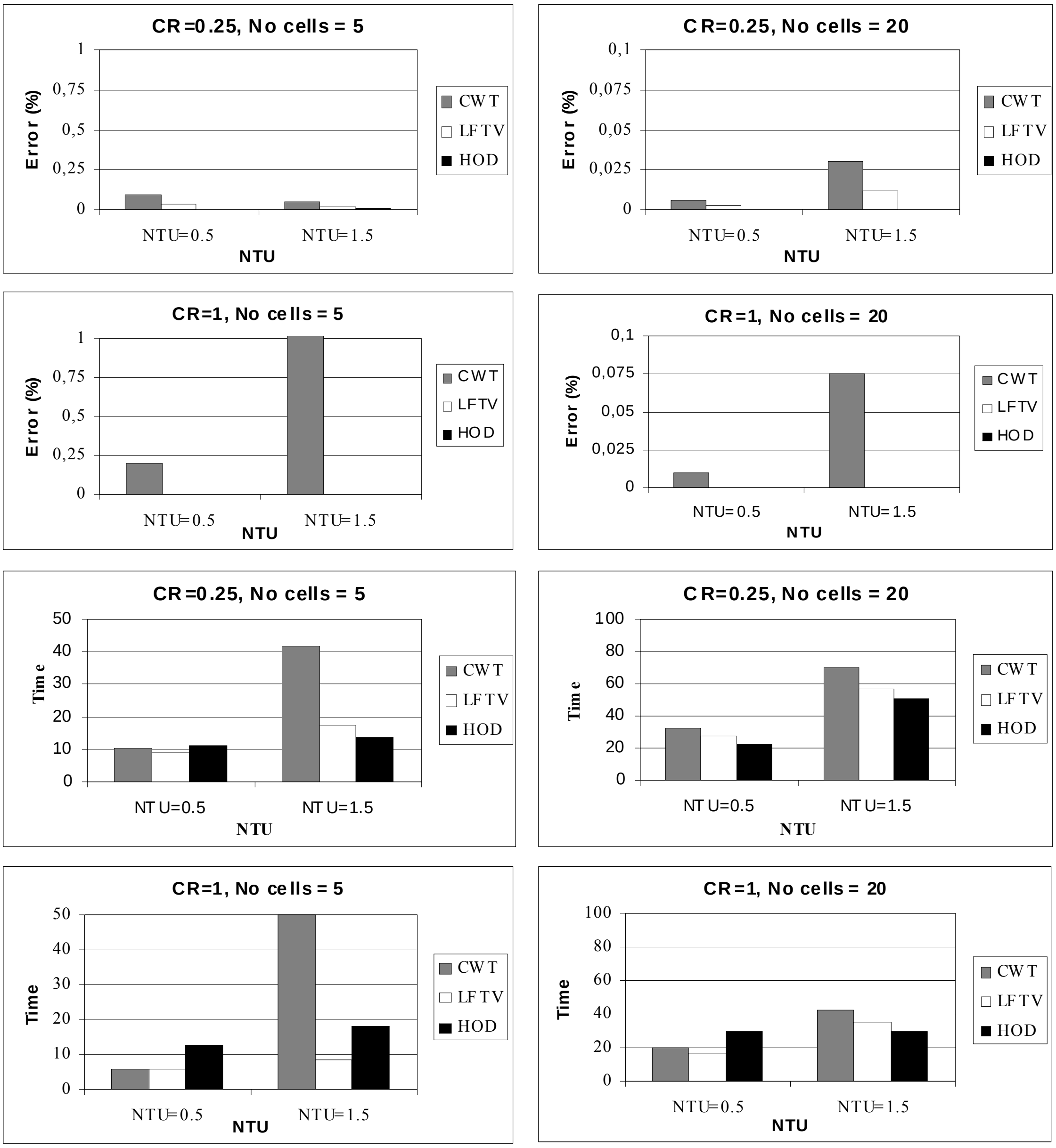

Figure 8. Final error and Time required to reach an error lower than 0.5 %, for a counter-current heat exchanger with: C_R = 0.25 and 1; NTU = 0.5 and 1.5; and for 5 x 1 (left) and 20 x 1 (right) calculation meshes.

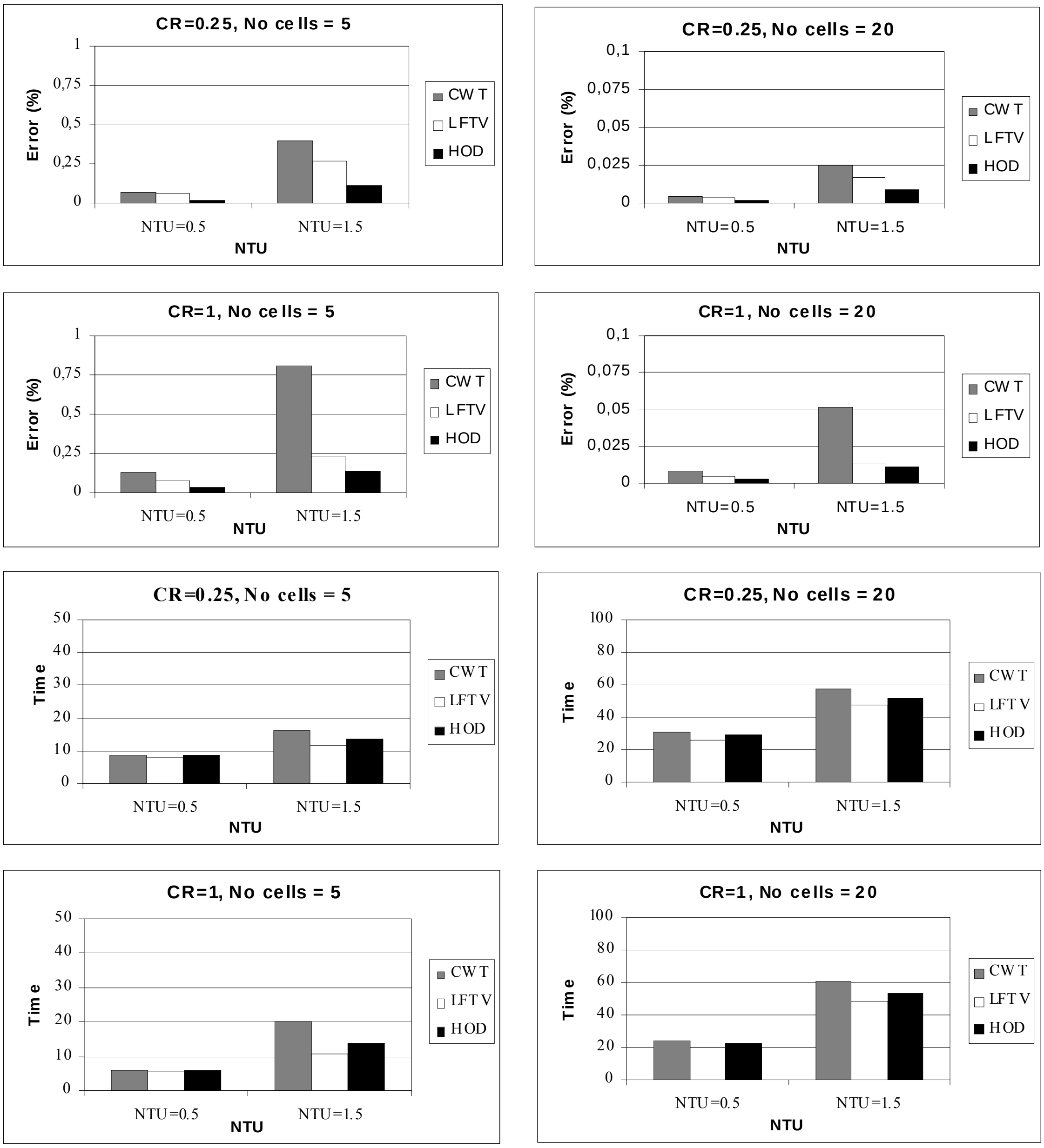

Figure 9. Final error and Time required to reach an error lower than 0.5 %, for a cross-flow heat exchanger with: C_R = 0.25 and 1; NTU = 0.5 and 1.5; and for 5 x 5 (left) and 20 x 20 (right) calculation meshes.