CPI, Edificio 8E, Cubo F, 5ª planta, Camino de Vera s/n, 46022 Valencia, Spain Tel. (34) 963877270, Fax (34) 963877272, Email: energeti@upvnet.upv.es

NUMERICAL CALCULATION OF MASS FLOW RATE IN CAPILLARY TUBES USING 'ART', AN ADVANCED SIMULATION SOFTWARE

José Miguel CORBERÁN [1], David FUENTES [2], José GONZÁLVEZ [3]

[1] Universidad Politécnica de Valencia, ETSII,
Valencia, Valencia, Spain
Phone: +34 96 387 73 23, Fax: +34 96 387 73 29, e-mail: corberan@ter.upv.es

[2] Universidad Industrial de Santander, Mechanical Engineering School,
Bucaramanga, Santander, Colombia
Phone: +57 7 634 63 76, e-mail: dafuedia@ter.upv.es

[3] Universidad Politécnica de Valencia, ETSII,
Valencia, Valencia, Spain
Camino de Vera s/n, Phone: +34 96 387 93 29,
Fax: +34 96 387 73 29, email: jgonzalv@ter.upv.es

ABSTRACT

A capillary tube is a common expansion device used in small sized refrigeration and air-conditioning systems. This paper present a numerical method used to calculate mass flow rate in a capillary tube. The proposed method solves the conservation laws (continuity, momentum and energy) in a 1D mesh. A iterative process is performed for a guessed value of the mass flow rate, and it is followed until critical flow conditions are obtained, the resulting length is compared with the capillary tube length and a new guess of the mass flow rate is imposed, and then, the iteration is repeated until convergence. In two phase flow, a separated flow model is assumed. Both, two-phase friction factor and viscosity models were determined by Lin correlation, and void fraction from Zivi correlation. This model is included in 'IMST-ART', a software for simulation and design of refrigeration equipment. The addition of capillary tube model allows to calculate the superheat at the evaporator giving the capillary tube geometry. A simulation with different operative conditions and capillary tube geometry is presented and the results are compared with those given by ASHRAE correlation.

1. INTRODUCTION

A capillary tube is a constant area expansion device used in a vapor-compression refrigeration system located between the condenser and the evaporator and whose function is to reduce the high pressure in the condenser to low pressure in the evaporator. The capillary tube expansion devices are widely used in refrigeration equipment, especially in small units such as household refrigerators, freezers and small air conditioners. Its simplicity is the most important reason to continue using it instead of other expansion devices. It is a long copper tube (1-6 m) with a very small inner diameter, often less than 1mm.

Many numerical and experimental investigations have been developed in order to study the refrigerant flow in capillary tubes. Normally, the numerical studies concerning capillary tubes found in the literature are targeted to the calculation of the mass flow rate through the tube for given geometry and operative conditions (Bittle et al. 1996, 1998, 2001, Wongwises 2000a, 2000b, Choi 2004, Melo 1996, Chang S. and Ro S.T.1996, Escanes F 1995, García-Valladares et al. 2002a,2002b, Liang and Wong 2001, Ro S.T., Kim S.G., Kim M.S.2002, among others).

The flow inside the capillary tube of a refrigeration system can be divided into a subcooled liquid region from the entrance to the point in which the fluid reaches saturated conditions, and a two-phase flow region after that point until the end of the capillary tube.

However, in reality, after the flow reaches the saturation pressure, there is a short region in which the evaporation does not start yet and the refrigerant becomes superheated, Chen et al.(1991), Chang and Ro (1996), Chen et al. (2001). The liquid in this region is not in thermodynamic equilibrium but under metastable condition. This region ends when quite suddenly the liquid starts to evaporate approaching thermodynamic two-phase equilibrium conditions. This metastable region is nevertheless quite short in comparison with the others two regions and it has been considered negligible in the present study. In any case, the authors have evaluated the influence of this effect in the evaluation of the mass flow rate through the capillary tube and have found that its influence is always lower than 5%, for the typical operating conditions of refrigeration systems. Additionally, that region presents serious difficulties in developing a correct physical modelling since the involved phenomena are very complex, (Dunn and Meyer 1998, Bittle et al. 2001), and, although some models are available, they are not of general application, depending on the tested fluid and on the geometry of the capillary tube.

Therefore, the authors have decided not to include it in the present model and concentrate on the development of a robust numerical scheme for the two-phase flow region, which is the region where the critical flow condition is encountered and therefore which plays the most important influence on the capillary tube performance.

2. GOVERNING EQUATIONS

The fundamental equations governing capillary tube flow are the mass, momentum and energy conservation equations. In a 1D mesh, these equations can be integrated and solved simultaneously by an iterative process for each control volume of length Δz like the shown in figure 1.

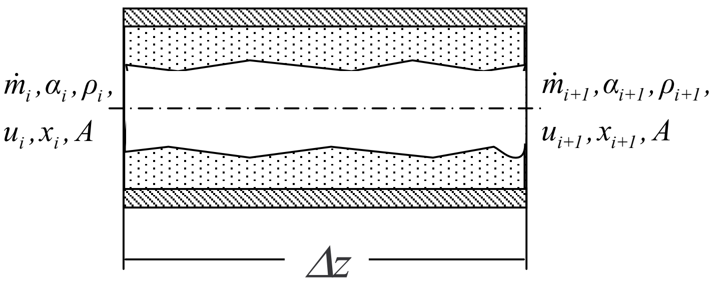


Figure 1. Control volume

The process can be repeated along the capillary tube length including single-phase and two-phase cells. For the two-phase flow region a separated flow model is assumed.

The model was developed with the following considerations:

- The capillary tube is horizontal (gravity effects are neglected), and of constant cross section.
- The flow in the capillary tube is steady, one-dimensional, and adiabatic.
- When the fluid reaches the saturation pressure, the fluid starts to evaporate.
- The fluid is always in local thermodynamic equilibrium corresponding to its local pressure.

Pressure losses can be defined by:

- Pressure drop by entrance effects (from the upstream tube to the capillary tube)
- Pressure drop by friction: Single-phase friction from entrance up to the saturation point, and two phase friction from saturation point up to the end.
- There is no recovery of the pressure in the enlargement from the outlet to the downstream tube, where the evaporation pressure is encountered.

2.1. Conservation equations

Mass conservation. The mass equation for the control volume shown in Figure 1 can be written as:

$$\left[\rho u A\right]_i^{i+1} = 0 \qquad (1)$$

or

$$\left[\frac{m}{A}\right]_i^{i+1} = [G]_i^{i+1} = 0 \tag{2}$$

where $[\ ]_i^{i+1}$ mean the variation from i to i+1 over the control volume of length Δz.

Momentum. Momentum equation can be written as:

$$[Gu]_i^{i+1} = \sum F \tag{3}$$

where ΣF is the balance of forces on the control volume given by $\Sigma\ F = F_f + F_{\Delta p}$ and F_f is the force by friction on tube's walls and $F_{\Delta p}$ is the force by difference of pressure on fluid. For single-phase flow, the one-dimensional momentum equation becomes:

$$[p]_i^{i+1} = \Delta p_{1\phi} = \left(f_{1\varphi}\frac{\Delta z}{d_h}\right)\frac{G^2}{2\rho_f} \tag{4}$$

where f_{1_φ} is the one-dimensional friction factor.

For two-phase flow, the momentum equation becomes:

$$-[p]_i^{i+1} = \Delta p_{2\phi} + \left\{ \left.\frac{x^2G^2}{\alpha\rho_g}\right|_i^{i+1} + \left.\frac{(1-x)^2G^2}{(1-\alpha)\rho_f}\right|_i^{i+1} \right\} \tag{5}$$

where the frictional pressure drop for two-phase flow is calculated following the known procedure $\Delta p_{2_\varphi} = \Delta p_{1_\varphi} * \Phi_{2_\varphi}$, with Φ_{2_φ} being a convenient two-phase multiplier.

Energy. The energy equation for adiabatic flow can be written as:

$$\left[h + \frac{u^2}{2}\right]_i^{i+1} = 0 \tag{6}$$

For single-phase one-dimensional flow it reduces to

$$[h]_i^{i+1} = 0 \tag{7}$$

and for two-phase flow to

$$\left\{[h_f + x(h_g - h_f)]G\right\}_i^{i+1} + \left\{G^3\left[\frac{(1-x)^3}{2(1-\alpha)^2\rho_f^2} + \frac{x^3}{2\alpha^2\rho_g^2}\right]\right\}_i^{i+1} = 0 \tag{8}$$

3. NUMERICAL PROCEDURE

The above system of equations (conservation equations + equations of state) constitutes a system of non linear equations with refrigerant properties, and liquid and vapor velocities as unknowns. The integration of the friction pressure drop for two-phase flow (Eq 5) is done by averaging its value between the corresponding inlet and outlet values:

$$\left|\frac{dp_{f_{1\varphi}}}{dz}\Phi_{2\varphi}\right| = \frac{1}{2}\left[\left(\frac{dp_{f_{1\varphi}}}{dz}\Phi_{2\varphi}\right)_i + \left(\frac{dp_{f_{1\varphi}}}{dz}\Phi_{2\varphi}\right)^{i+1}\right] \tag{9}$$

The calculation of the capillary tube performance for given operating conditions (upstream and downstream) involves the calculation of the mass flow rate and the evolution of the refrigerant properties all along the tube. For a given capillary tube geometry (inner diameter and length) and given operative conditions, the whole problem becomes fully implicit in the mass flow rate and in the outlet refrigerant properties. Therefore, the mass flow is selected as main iteration parameter (unknown).

For a given guess of the mass flow rate, the calculation starts from the inlet section and proceeds till the end of the capillary tube. Integrating the conservation equations over Δz, for non critical conditions, the calculated capillary tube length must be the real capillary tube length. Then, a Newton-Raphson like iteration process must start again with a new adequate value of the mass flow rate until the real capillary tube length is obtained. Otherwise, critical flow is present and the iterative process is performed so that the critical condition should be attained just at the outlet section of the tube

For a guessed mass flow rate, the process start calculating the entrance pressure from the expression found in Hewitt(1994). If flow regime after to entrance has a subcooled condition the length needed to the fluid reach the saturation condition (L_{sp}) is calculated by:

$$L_{1\varphi} = \frac{2\Delta p_l d_c \rho_f}{f_{1\varphi} G^2} \tag{10}$$

where Δp_l is the difference between the liquid pressure just after to entrance effect and the pressure corresponding to the onset of vaporization.

Once the saturation point is reached, the calculation proceeds evaluating the two-phase cells, and an iterative process must be followed in order to get the correct values of the fluid properties at the outlet section of the cell. With Δp given, the cell length (Δz) is calculated by, :

$$\Delta z = \frac{-[p]_i^{i+1} - \left\{ \left. \frac{x^2 G^2}{\alpha \rho_g} \right|_i^{i+1} + \left. \frac{(1-x)^2 G^2}{(1-\alpha)\rho_f} \right|_i^{i+1} \right\}}{\left| \left. dp_{f_{1\varphi}} \middle/ dz \right. \Phi_{2\varphi} \right|} \tag{11}$$

Being the Eq. (11) implicit on the quality.

And, the enthalpy at the exit of the cell can be calculated by:

$$h_{i+1} = h_i + \left\{ G^3 \left[\frac{(1-x)^3}{2(1-\alpha)^2 \rho_f^2} + \frac{x^3}{2\alpha^2 \rho_g^2} \right] \right\}_i^{i+1} \tag{12}$$

Then, the quality at the outlet of the cell can be calculated. The process is repeated until vapour quality converges. At the completion of a successful iteration the right exit pressure or the cell length are known.

At the completion of a successful global iteration, the calculated capillary length is equal to the actual length, the mass flow rate has been calculated, and p_{exit} is known. If $p_{exit} > p_{evap}$ critical flow exists, otherwise, $p_{exit} = p_{evap}$ and flow is non critical.

A complete and detailed explanation of the method can be found in Fuentes (2004).

3. EMPIRICAL COEFFICIENTS

The mathematical model requires some additional information generally obtained from empirical correlations. The following sources has been used for the present study.

The *single phase flow* friction factor is evaluated from the expressions proposed by Churchill (taken from by Lin et al. (LIN1991)). The *two phase flow* frictional pressure drop is evaluated from the expressions proposed by Lin et al. (LIN1991) and the *void fraction* α parameter is evaluated by Zivi's correlation (taken from WALLIS1969).

4. ASHRAE CORRELATION

The ASHRAE correlation (ASHRAE 2002) is a generalized correlation for calculating mass flow rate in capillary tube. Based on test with R134a, R22 and R410A, Wolf et al. (1995) developed a general method for predicting refrigerant mass flow rate through a capillary tube. The operating parameters considered are inlet pressure P_{in}, degree of subcooling ΔT_{sub}, and saturation pressure P_{sat} corresponding to inlet temperature. The existence of the metastable region is represented as a function of surface tension, pressure drop, and surface geometry with which the liquid is in contact. In addition to these variables, critical temperature T_c and heat of vaporization h_{fg} are also included to dimensionless inlet subcooling ΔT_{sub} and to consider potential effect of vapor bubble formation and growth, respectively. Capillary tube diameter d and length L are included to consider geometric effects on mass flow rate. Surface roughness of inner wall affects the friction loss, the inception of flash, and the mass flow rate. However, the surface roughness is not included in the dimensionless parameter because the commercial tubes are available only in a limited band of surface roughness. The final form for each parameter is shown in the table 1.

The mass flow rate for a inlet subcooled condition is calculated by the Equation:

$$\pi_8 = 1.8925 * \pi_1^{-0.484} * \pi_2^{-0.824} * \pi_4^{1.369} * \pi_5^{0.0187} * \pi_6^{0.773} * \pi_7^{0.265} \tag{13}$$

And for a inlet quality condition by:

$$\pi_8 = 187.27 * \pi_1^{-0.635} * \pi_2^{-0.189} * \pi_4^{0.645} * \pi_5^{-0.163} * \pi_6^{-0.213} * \pi_7^{-0.483} \tag{14}$$

Table 1. Dimensionless Pi-groups

Pi-groups	Formula	Effect
π_1	L/d	Geometry
π_2	$\frac{h_{fg}\rho_f^2 d^2}{\mu_f^2}$	Vaporization effect
π_3	$\frac{\sigma\rho_f d}{\mu_f^2}$	Bubble formation
π_4	$\frac{P_{in}\rho_f d^2}{\mu_f^2}$	Inlet pressure
π_5	$d^2 c_p \Delta T_{sc}\rho_f^2 / \mu_f^2$	Inlet condition
π_5	x	Inlet condition
π_6	ρ_f / ρ_g	Friction, Bubble growth
π_7	$(\mu_f - \mu_g) / \mu_g$	Viscous effect
π_8	$\dot{m} / d\mu_f$	Flow rate

4. ART PROGRAM

ART is a software package recently developed for the analysis and optimization of refrigeration system equipment based on the vapor compression principle. ART is able to calculate a complete refrigeration system, including the detailed modeling of the HEs, compressor, pipes lines, Corberán (2002). Now, it incorporates the two models above mentioned to calculate the mass flow rate through capillary tubes. With this new component, ART is able to predict the produced evaporator superheat when the system operates in a particular condition for a given capillary tube geometry.

5. RESULTS AND CONCLUSIONS

The described capillary tube model, has been validated with experimental data from Wolf et al. (1995). The results of the mass flow rate calculated using the presented model respected to Bittle experimental data are shown in the Figure (2). The mean error of the calculated mass flow rate respect to all the experimental data obtained from the cited reference is around of 2.3% and the RMS is around 5.7%. As can be observed in Figure (2) the model has a good agreement for Wolf et al. experimental data.

Also, the presented capillary tube model is validated using the commercial ART program. A 2kW split air conditioning system, working with R22 and with indoor temperature equal to 23ºC and outdoor temperature equal to

35 ºC is simulated. The capillary tube is 1 m long and its diameter has 1.5 mm. Three different studies are presented: the first one varying the capillary tube length; the second one, varying the indoor temperature; and the third one varying the outdoor temperature. Theses studies are performed calculating mass flow rate through the capillary tube by the presented numerical model and ASHRAE equations.

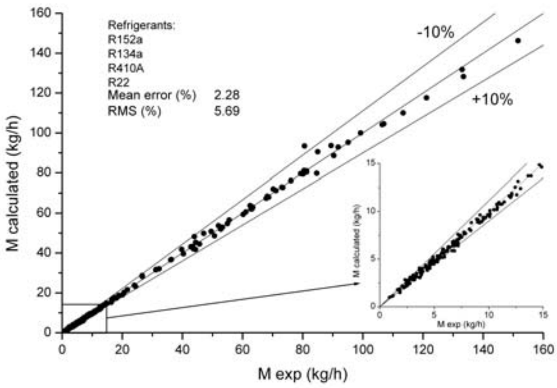


Figure 2. Comparison of calculated results with the experimental data from Bittle.

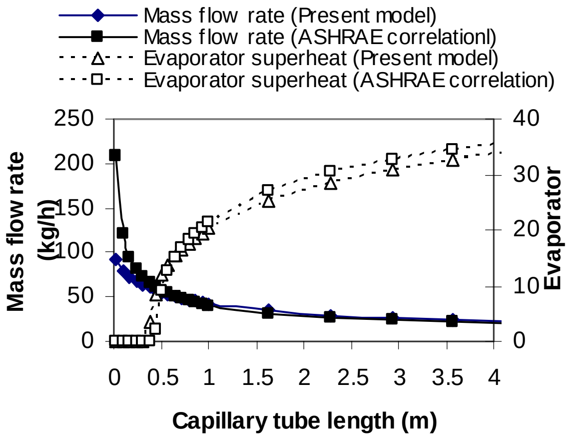


Figure 3. Capillary tube length influence over refrigerant mass flow rate and evaporator superheat.

The Figure 3 show the influence of the capillary tube length over the refrigerant mass flow rate and evaporator superheat. As can be observed, the ART program is able to predict the minimum length needed which produce a superheat in the evaporator. In this case, the zero superheat value finished near to the capillary tube length equal to 0.4 m. Additionally, as can be observed, the calculated superheat at the evaporator and mass flow rate, obtained from evaluating the capillary tube behavior with the numerical method, are close values to those calculated using the ASHRAE equations, indicating a good adjustment respected to the ASHRAE equations.

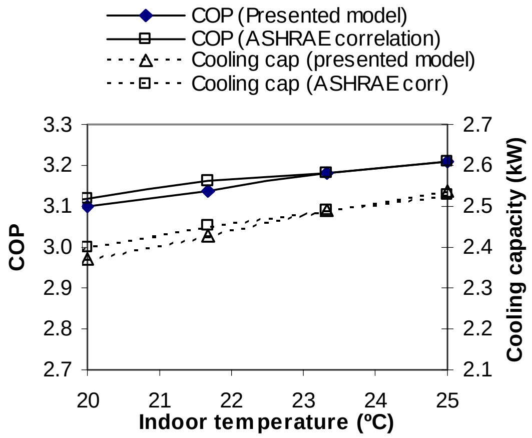
Fixing the capillary tube length to 1 m, the influence of the indoor and outdoor temperature over COP and cooling capacity is shown in the Figure 4 and 5, respectively. For the indoor temperature variation, the ART program calculates close values for COP and cooling capacity when the capillary tube is calculated by the numerical model or ASHRAE correlation. A similar characteristic is observed for outdoor temperature variation. Also, as can be observed, in both case, the variation of the COP and cooling capacity has a correct tendency.

CONCLUSIONS

A numerical model for calculating mass flow rate through capillary tubes working with pure refrigerants or refrigerant mixtures has been developed by means of the integration of the conservation equations (mass, momentum and energy) over individual control volumes. The model include non critical and critical flow conditions. The model assume thermodynamic equilibrium and one-dimensional two-phase flow. The metaestable condition is not taken into account.

The ART program is able to predict the minimum capillary tube length in order to get a net evaporator superheat. Also, the addition of capillary tube models allows calculating the superheat at the evaporator giving the capillary tube geometry. In general, a very good agreement between numerical model and ASHRAE correlation has been observed and the method presents continuous and monotone solutions with the variation of inlet and operating conditions.

Because the numerical model is based on the applications of physical laws, it is possible to extrapolate with greater confidence than ASHRAE correlation to other operating conditions and other refrigerants.



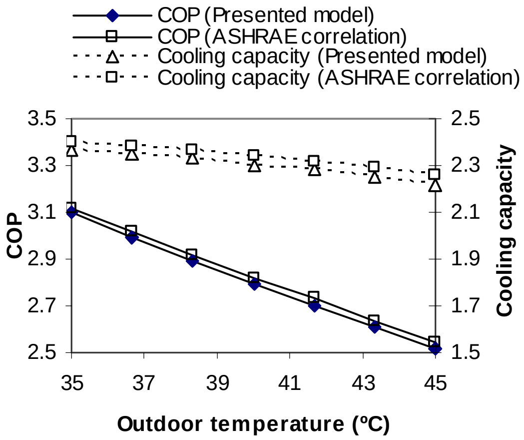


Figure 4. Indoor temperature influence over COP and Cooling capacity.

Figure 5. Outdoor temperature influence over COP and cooling capacity.

NOMENCLATURE

A	Cross flow area	(m^2)	**Subscripts**	
C	Constants	(dimensionless)	c	contraction
d	Diameter	(m)		capillary tube
F	Force	(N)	exit	at outlet section
f	Friction factor	(dimensionless)	exp	experimental
G	Mass flux	($kg\ s^{-1}\ m^{-2}$)	evap	evaporator
h	Specific enthalpy	($kJ\ kg^{-1}$)	1φ	single-phase
L	Capillary tube length	(m)	2 φ	two-phase
u	Velocity	($m\ s^{-1}$)	i	inlet of the cell
p	Pressure	(Pa)	i+1	outlet of the cell
T	Temperature	(K)	w	wall
$\dot{m}$	mass flow rate	($kg\ s^{-1}$)	f	liquid (at sat.
x	Quality	(dimensionless)		condition)
z	Horizontal distance	(m)	f	friction
α	Void fraction	(dimesionless)	g	vapor (at sat.
Δ	difference of			condition)
Φ	Factor	(dimesionless)	sub	subcooling
ρ	Density	($kg\ m^{-3}$)	in	inlet
μ	Viscosity	(Pa s)	fg	vaporization
σ	ration of flow areas	(dimensionless)	sat	saturation

REFERENCES

ASHRAE, 2002, Adiabatic capillary tube selection, *Refrigeration Handbook*, ch. 45, pp.45.26-45.30, ASHRAE.

Bittle R., et al., 1998, A Generalized performance prediction method for adiabatic capillary tubes, *HVAC&Research*, vol 4, no. 1, p27-44.
Bittle R.R., Oliver J.V., Carter J.A., 2001, Extended Insights into the metastable liquid region behavoir in an adiabatic capillary tube, *HVAC&R Research*, vol 7, no. 2, p107-123.
Bittle R., Pate M., 1996, A theorical model for predicting adiabatic capillary tube performance with alternative refrigerants, *ASHRAE Trans.&Research*, vol 102, no. 2, p52-64.
Chang S. and Ro S.T., 1996, Experimental and numerical studies on adiabatic flow of HTC mitures in capillary tubes, *Int. Ref. Conf. Proceedings*, Purdue University, p83-88.
Chen D., Sui L., 2001, Underpressure of vaporization of refrigerant R134a through a diabatic capilary tube, *Int. J. of Refrigeration*, vol 24, p261-271.
Chen Z.H., et al., 1991, A correlation for metastable flow of refrigerant 12 through capillary tube, *ASHRAE Transactions*, vol 96, p550-554.
Choi J., et al (2004), An empirical correlation and rating charts for the performance of adiabatic capillary tubes with alternative refrigerants, *Applied Thermal Engineering*, vol 24, p29-41.
Corberán, J.M. et al., 2002, 'ART' A computer code to assist the design of refrigeration and A/C equipment, *Int. Ref. Conf. Proceedings*, Purdue University, R9-4.
Domanski P, Didion D., 1983, *Computer Modeling of the vapor compresion cycle with constant flow area expansion device*. Building Science Series, National Bureau of Standars of USA.
Dunn W., Meyer J.J., 1998, New Insights into the behavoir of the metastable region of the operating capillary tybe, *HVAC&R Research*, vol 4, no.1, p105-115.
Escanes F., et al., 1995, Numerical simulation of capillary-tube expansion device, *Int. J. of Refrigeration*, vol 18, no. 2, p113-122.
Fuentes D, et al., 2004, Numerical calculation of critical mass flow rate in adiabatic capillary tubes. *3rd Int. Sym. On Two-Phase Flow modeling and experimentation*, Pisa 22-24 September 2004. Paper in review.
García Valladares O, et al., 2002a, Numerical simulation of capillary expansion device behavoir with pure and mixed refrigerants considerating metaestable region, Part I: mathematical formulation and numerical model, *Applied Thermal Engineering,* vol 22, p 173-182.
García Valladares O, et al. 2002b, Numerical simulation of capillary expansion device behavoir with pure and mixed refrigerants considerating metaestable region, Part II: experimental validation and parametrics studies, *Applied Thermal Engineering,* vol 22, p 379-391.
Hewitt G.F., Shires G.L., Bott~T.R., 1994, *Process Heat Transfer*, CRC Press, section 10.4.2.
Liang S.M., Wong T.N., 2001, Numerical modeling of two-phase refrigerant flow through adiabatic capillary tubes, *Applied Thermal Engineering*, vol 21, p1035-1048.
Lin S. et al, 1991, Local Frictional pressure drop during vaporization of R-12 through capillary tubes, *Int. J. Multiphase Flow*, vol 17, no. 1, p95-102.
Melo C, Mezavila M.,1996, CAPHEAT: An Homogeneous Model to simulate refrigerant flow through non-adiabatic capillary tubes, *Int. Ref. Conf. Proceedings*, Purdue University, p95-100.
Ro S.T., Kim S.G., Kim M.S., 2002, Experimental investigation of the performance of R22, R407C and R410A in - several capillary tubes for air-conditioners, *Int. J. of Refrigeration*, vol 25, p521-531.
Wallis, G., 1969, *One-Dismensional Two-phase flow*, McGraw-Hill.
Wolf D.A., et al. 1995, Capillary tube performance with alternative refrigerants, *Tech. Report 762, ASHRAE*.
Wongwises S., Chan P., 2000a, Two-phase separated flow model of refrigerant flowing through capillary tubes, *Int. Comm. Heat Mass Transfer*, vol 27, no 3, p343-356.
Wongwises S., et al, 2000b, A comparison of the flow characteristics of refrigerants flowing through adiabatic capillary tubes, *Int. Comm. Heat Mass Transfer,* vol 27, no. 5, p611-621.

ACKNOWLEDGEMENT

This research has been partially funded by the Spanish Ministry for Science and Technology (MCYT) through the project SEAAUTO (DPI2001-2682) with FEDER (EC) funds contribution. The authors gratefully acknowledge their financial support.